Lesson 17: The Area of a Circle

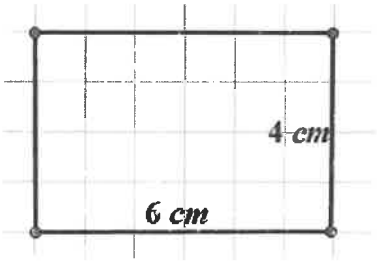
Classwork

Exercises 1–3

Solve the problems below. Explain your solution.

1. Find the radius of a circle if its circumference is 37.68 inches. Use $\pi \approx 3.14$.

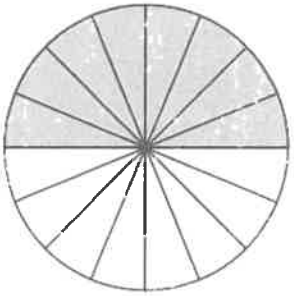
2. Determine the area of the rectangle below. Name two ways that can be used to find the area of the rectangle.



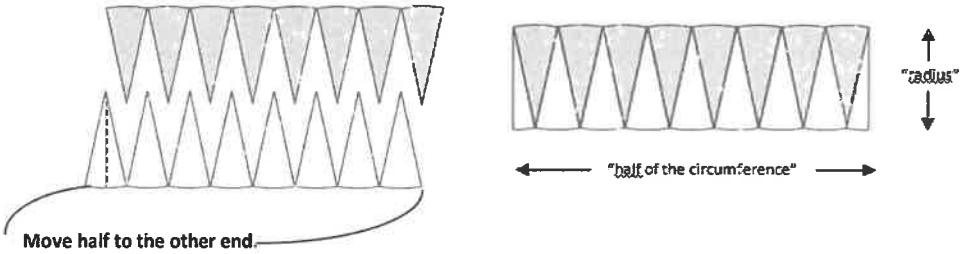
3. Find the length of a rectangle if the area is 27 cm^2 and the width is 3 cm.

Exploratory Challenge

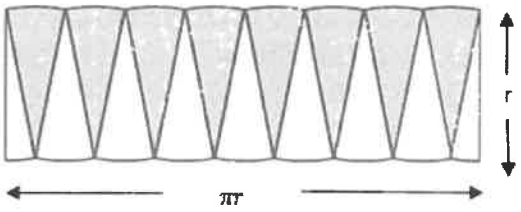
To find the formula for the area of a circle, cut a circle into 16 equal pieces.



Arrange the triangular wedges by alternating the "triangle" directions and sliding them together to make a "parallelogram." Cut the triangle on the left side in half on the given line, and slide the outside half of the triangle to the other end of the parallelogram in order to create an approximate "rectangle."



The circumference is $2\pi r$, where the radius is r. Therefore, half of the circumference is πr.



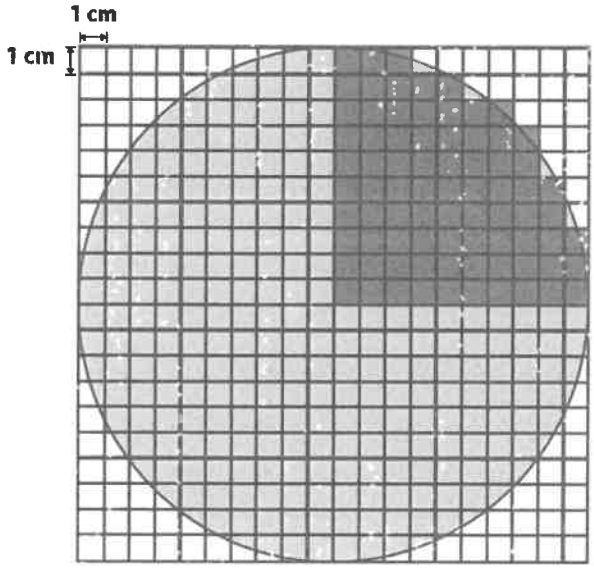
What is the area of the "rectangle" using the side lengths above?

Are the areas of the "rectangle" and the circle the same?

If the area of the rectangular shape and the circle are the same, what is the formula for the area of the circle?

Example 1

Use the shaded square centimeter units to approximate the area of the circle.



What is the radius of the circle?

What would be a quicker method for determining the area of the circle other than counting all of the squares in the entire circle?

Using the diagram, how many squares were used to cover one-fourth of the circle?

What is the area of the entire circle?

Example 2

A sprinkler rotates in a circular pattern and sprays water over a distance of 12 feet. What is the area of the circular region covered by the sprinkler? Express your answer to the nearest square foot.

Draw a diagram to assist you in solving the problem. What does the distance of 12 feet represent in this problem?

What information is needed to solve the problem?

***When finding an exact answer, it will be written in terms of pi. For example, what would be the exact answer to the problem above?

Example 3

John is making a circular table out of a square piece of wood. The radius of the circle that he is cutting is 3 feet. How much waste will he have for this project? Express your answer to the nearest square foot.

Draw a diagram to assist you in solving the problem. What does the distance of 3 feet represent in this problem?

What information is needed to solve the problem?

What information do we need to determine the area of the square and the circle?

How will we determine the waste?

Exercises 4–6

4. A circle has a radius of 2 cm.
   a. Find the exact area of the circular region.

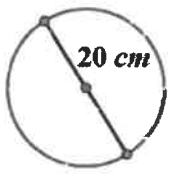
   b. Find the approximate area using 3.14 to approximate π.

5. A circle has a radius of 7 cm.
   a. Find the exact area of the circular region.

   b. Find the approximate area using $\frac{22}{7}$ to approximate π.

   c. What is the circumference of the circle?

6. Joan determined that the area of the circle below is 400π cm^2. Melinda says that Joan's solution is incorrect; she believes that the area is 100π cm^2. Who is correct and why?



Lesson 18: More Problems on Area and Circumference

Classwork

Opening Exercise

Draw a circle with a diameter of 12 cm and a square with a side length of 12 cm. Determine the area of the square and the circle.

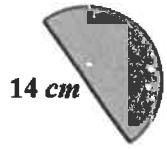
Find the area of half of the square and half of the circle.

What is the ratio of the new area to the original area for the square and for the circle?

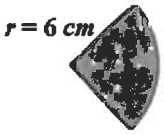
Find the area of one-fourth of the square and one-fourth of the circle. What is the ratio of the new area to the original area for the square and for the circle?

Example 1

Find the area of the following semicircle. Use $\pi \approx \frac{22}{7}$.



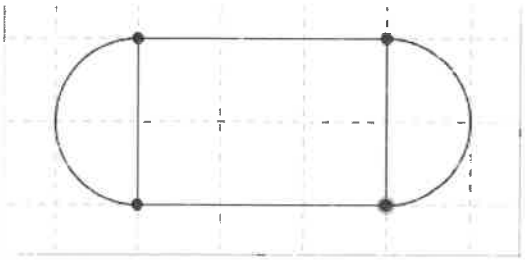
What is the area of the quarter circle? Use $\pi \approx \frac{22}{7}$.



Example 2

Marjorie is designing a new set of placemats for her dining room table. She sketched a drawing of the placement on graph paper. The diagram represents the area of the placemat consisting of a rectangle and two semicircles at either end. Each square on the grid measures 4 inches in length.

Find the area of the entire placemat. Explain your thinking regarding the solution to this problem.



If Marjorie wants to make six placemats, how many square inches of fabric will she need? Assume there is no waste.

Marjorie decides that she wants to sew on a contrasting band of material around the edge of the placemats. How much band material will Marjorie need?

Example 3

The circumference of a circle is 24π cm. What is the exact area of the circle?

Draw a diagram to assist you in solving the problem.

What information is needed to solve the problem?

Next, find the area.

Exercises

1. Find the area of a circle with a diameter of 42 cm. Use $\pi \approx \frac{22}{7}$.

2. The circumference of a circle is 9π cm.
   a. What is the diameter?

   b. What is the radius?

   c. What is the area?

3. If students only know the radius of a circle, what other measures could they determine? Explain how students would use the radius to find the other parts.

4. Find the area in the rectangle between the two quarter circles if $AF = 7$ ft, $FB = 9$ ft, and $HD = 7$ ft. Use $\pi \approx \frac{22}{7}$. Each quarter circle in the top-left and lower-right corners have the same radius.