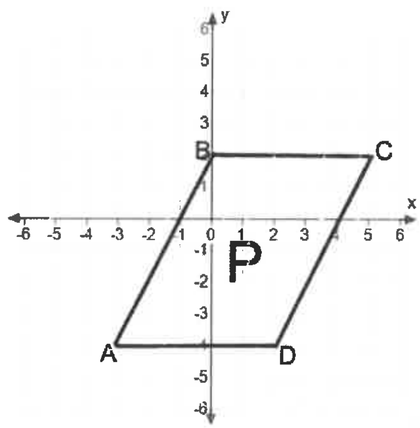
Lesson 19: Unknown Area Problems on the Coordinate Plane

Classwork

Example: Area of a Parallelogram

The coordinate plane below contains figure P, parallelogram $ABCD$.



a. Write the ordered pairs of each of the vertices next to the vertex points.

b. Draw a rectangle surrounding figure P that has vertex points of A and C. Label the two triangles in the figure as S and T.

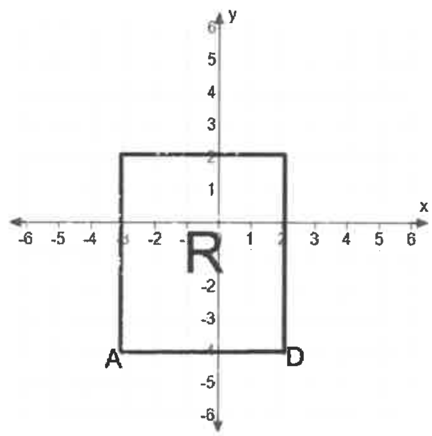
c. Find the area of the rectangle.

d. Find the area of each triangle.

e. Use these areas to find the area of parallelogram $ABCD$.

The coordinate plane below contains figure R, a rectangle with the same base as the parallelogram above.

f. Draw triangles S and T and connect to figure R so that you create a rectangle that is the same size as the rectangle you created on the first coordinate plane.

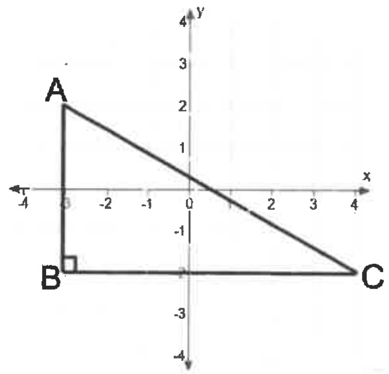


g. Find the area of rectangle R.

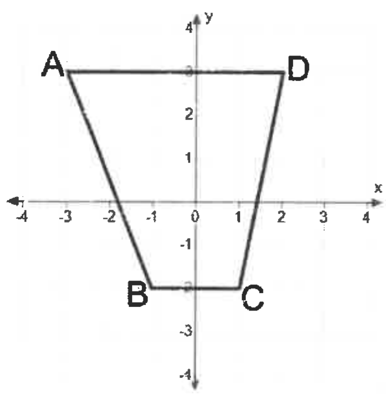
h. What do figures R and P have in common?

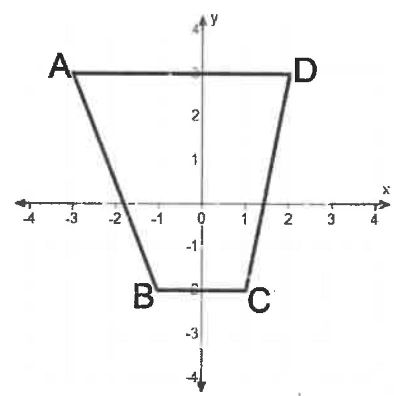
Exercises

1. Find the area of triangle ABC.

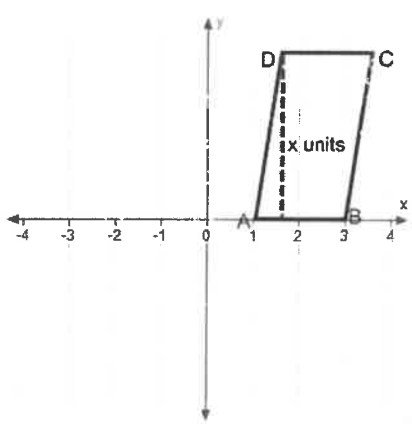


2. Find the area of quadrilateral $ABCD$ two different ways.

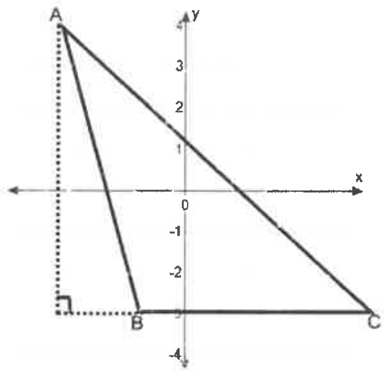




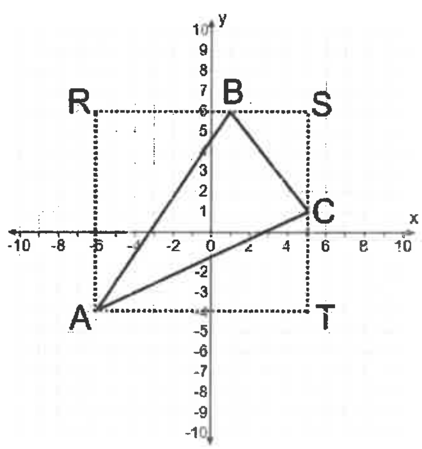
3. The area of quadrilateral $ABCD$ is 12 sq. units. Find x.



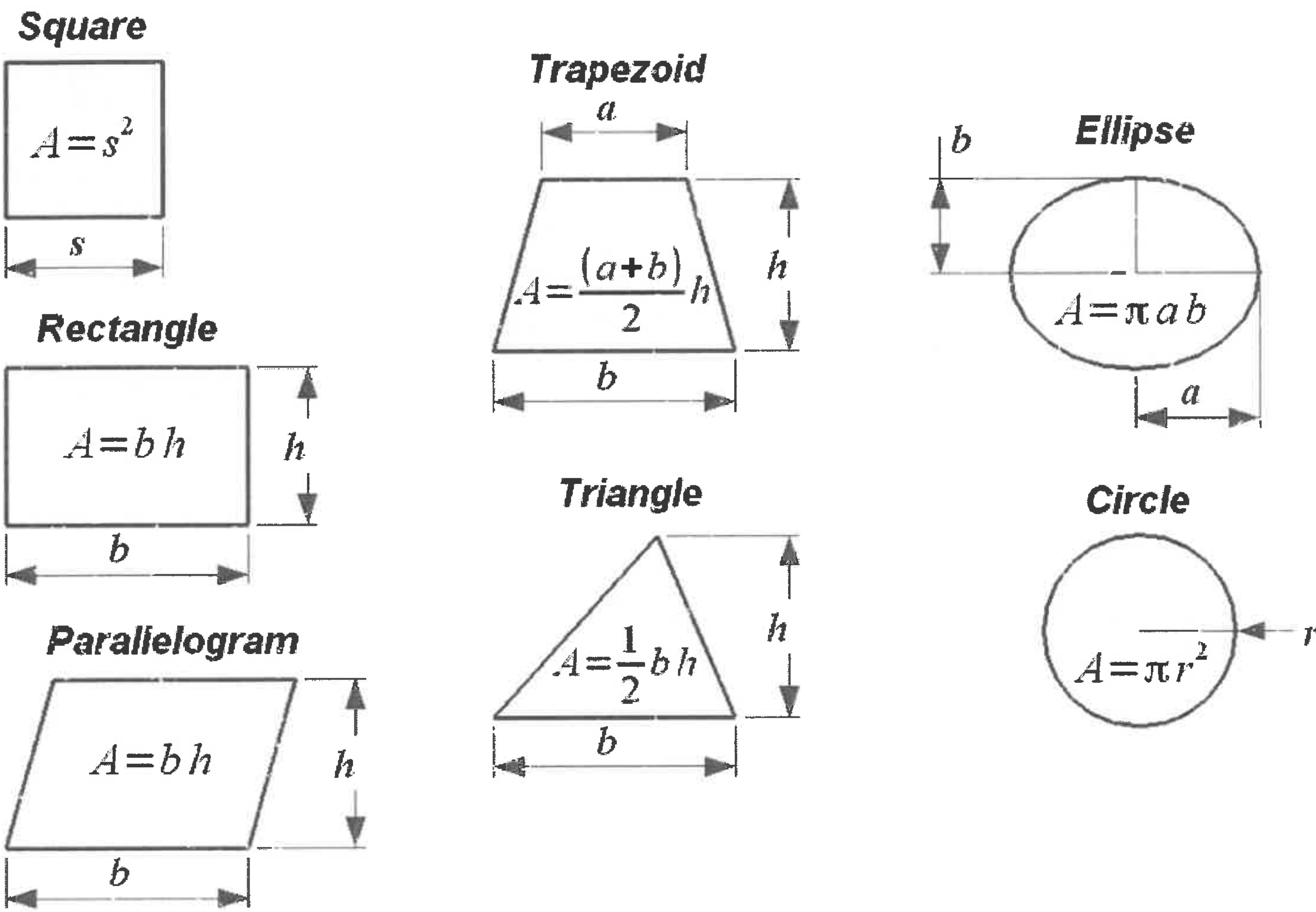
4. The area of triangle ABC is 14 sq. units. Find the length of side $\overline{BC}$.



5. Find the area of triangle ABC.



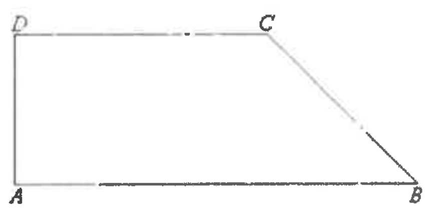
Area Formulas for Basic Figures



Lesson 20: Composite Area Problems

Composite Figure: A ______________________ (or shape) that can be ______________________ into _______ or _________ basic figures.

Examples of Composite Figures:

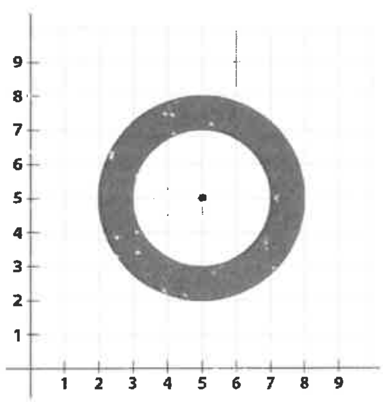



Composite shape

Classwork

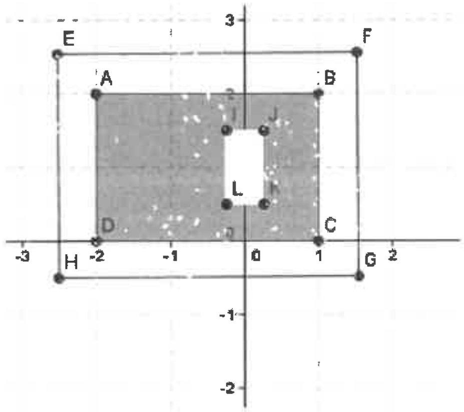
Example 1

Find the composite area of the shaded region. Use 3.14 for π.



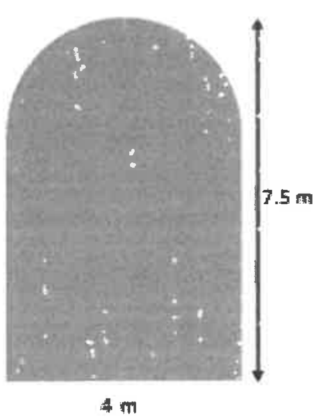
Exercise 1

A yard is shown with the shaded section indicating grassy areas and the unshaded sections indicating paved areas. Find the area of the space covered with grass in units^2.



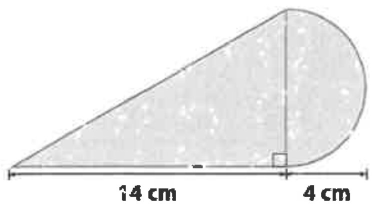
Example 2

Find the area of the figure that consists of a rectangle with a semicircle on top. Use 3.14 for π.



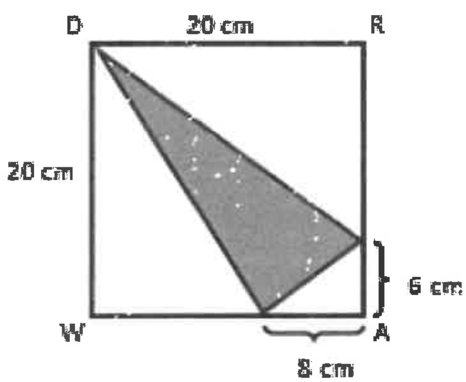
Exercise 2

Find the area of the shaded region. Use 3.14 for π.



Example 3

Find the area of the shaded region.



What recognizable shapes are in the figure?

____

What else is created by these two shapes?

____

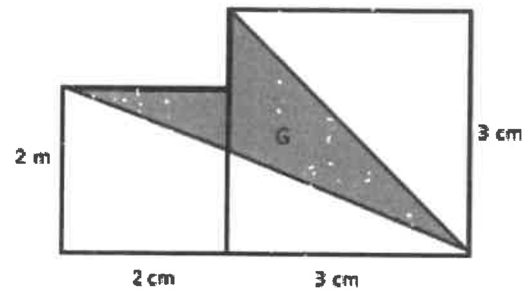
What specific shapes comprise the square?

____

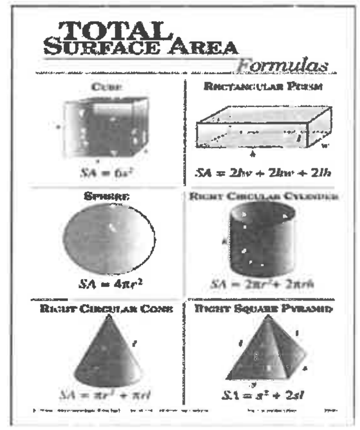
Redraw the figure separating the triangles; then, label the lengths.

Exercise 3

Find the area of the shaded region. The figure is not drawn to scale.

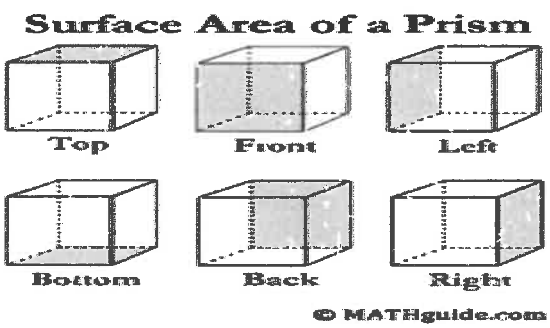


Lesson 21: Surface Area – Words to Know

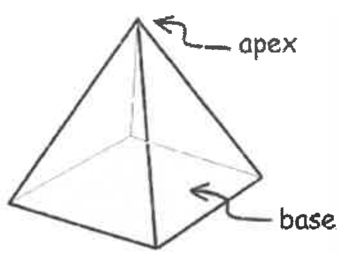


Surface Area: The __________ of the _____________ of all the _____________________.

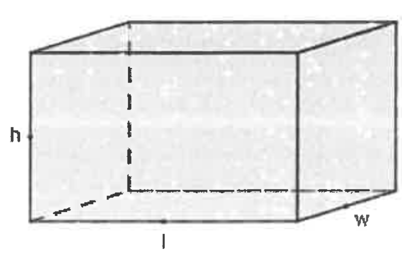
Face: The _________ _____________________ of a _________ figure.



Base: The _________________ a _______________ object _________________ on.



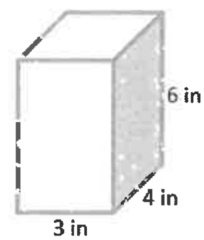
Right Prism: Solid with _______ "end" _________________ (called its _______________) that are exact _______________ of each other.

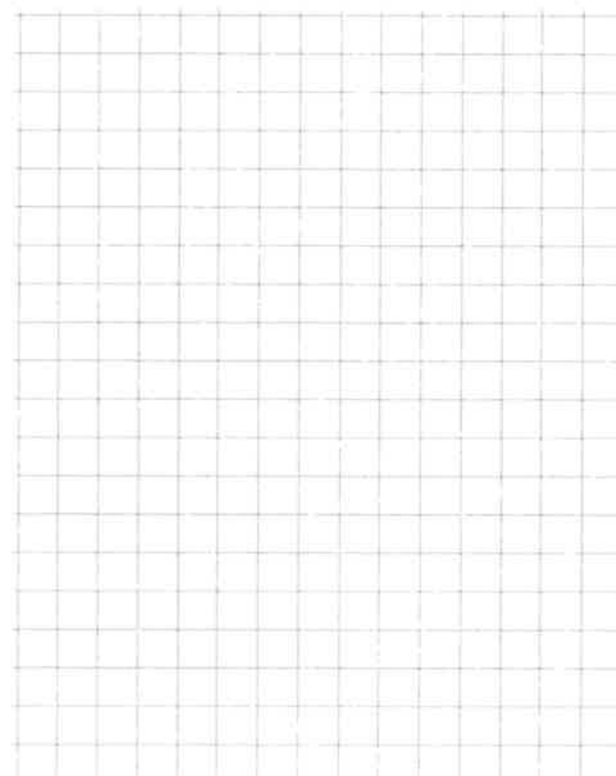


Classwork

Opening Exercise: Surface Area of a Right Rectangular Prism

On the provided grid, draw a net representing the surfaces of the right rectangular prism (assume each grid line represents **1** inch). Then, find the surface area of the prism by finding the area of the net.





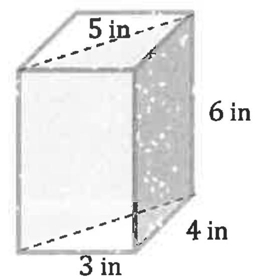
What other ways could we have found the surface area of the rectangular prism?

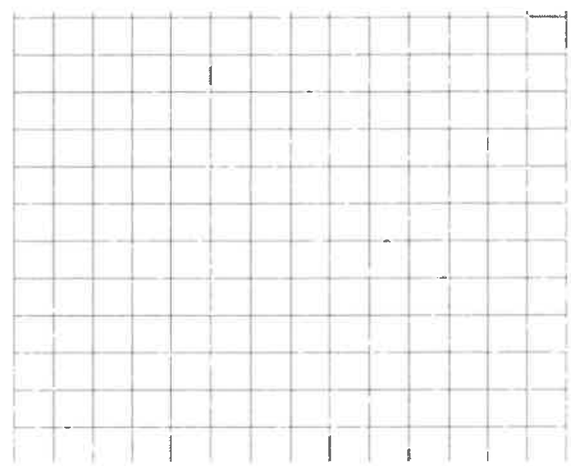
S.A. Formula of Rectangular **Prism:** ______________________________

Exercise 1

Marcus thinks that the surface area of the right triangular prism will be half that of the right rectangular prism and wants to use the modified formula

$SA = \frac{1}{2}(2lw + 2lh + 2wh)$. Do you agree or disagree with Marcus? Use nets of the prisms to support your argument.





Discussion

- The surface area formula for a right rectangular prism cannot be applied to a right triangular prism. Why?

  ____

  ____

- If you compare the nets of the right rectangular prism and the right triangular prism, what do the nets seem to have in common? (Hint: what do ***all*** right prisms have in common?)

  ____

  ____

- What is a lateral face?

____

____

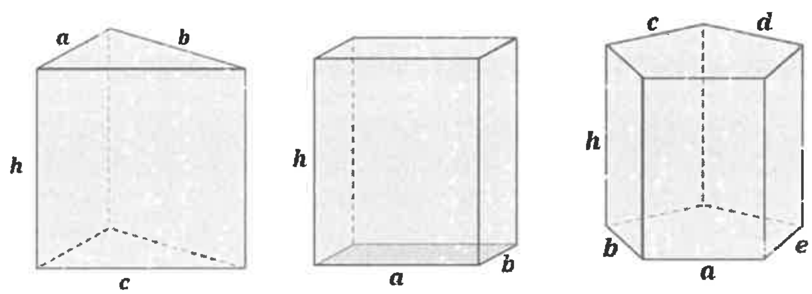
- What is the lateral area?

____

____

Example 1: Lateral Area of a Right Prism

A right triangular prism, a right rectangular prism, and a right pentagonal prism are pictured below, and all have equal heights of h.



a. Write an expression that represents the lateral area of the right triangular prism as the sum of the areas of its lateral faces.

b. Write an expression that represents the lateral area of the right rectangular prism as the sum of the areas of its lateral faces.

c. Write an expression that represents the lateral area of the right pentagonal prism as the sum of the areas of its lateral faces.

d. What value appears often in each expression and why?

e. Rewrite each expression in factored form using the distributive property and the height of each lateral face.

f. What do the parentheses in each case represent with respect to the right prisms?

g. How can we generalize the lateral area of a right prism into a formula that applies to all right prisms?

*****This is where lateral area comes from. It can now be part of the surface area formula for ALL right prisms.**

Lesson Summary

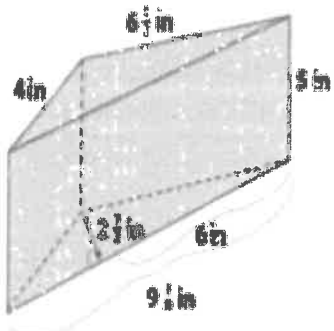
The surface area of a right prism can be obtained by adding the areas of the lateral faces to the area of the bases. The formula for the surface area of a right prism is $SA = LA + 2B$, where SA represents the surface area of the prism, LA represents the area of the lateral faces, and B represents the area of one base. The lateral area LA can be obtained by multiplying the perimeter of the base of the prism times the height of the prism.

Practice:

$$LA = P \cdot h \qquad SA = LA + 2B$$

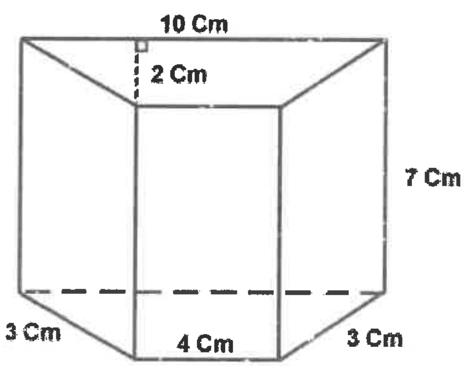
Find the surface area using the formula above.

1)



Find the surface area by finding the sum of the area of the faces.

2)

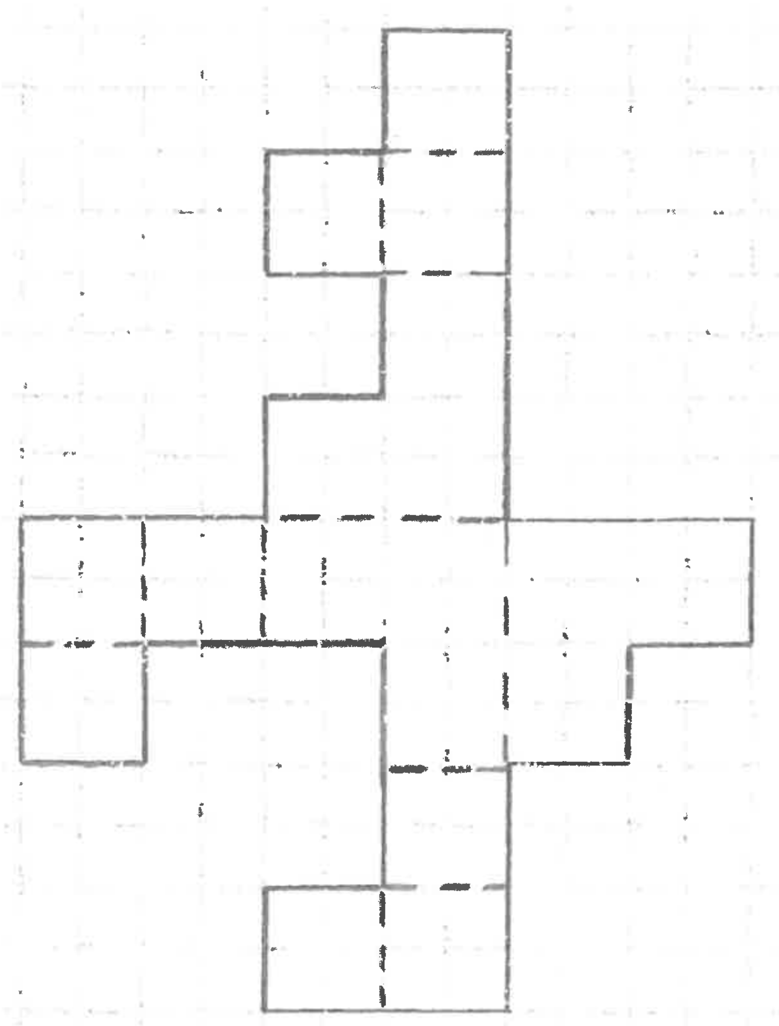


Lesson 22: Surface Area

Classwork

Opening Exercise

What is the area of the composite figure in the diagram? Cut out the net, and see if it can be folded into a three-dimensional figure.



Area = ____________________

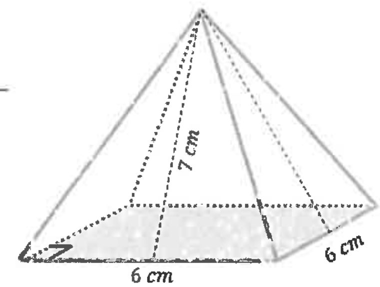
Example 1

What shape is the base? ______________________

What shape are the lateral faces? ____________________ How many? ________

What is the area formula of a triangle? ____________________

Classify the three-dimensional figure. ____________________________



Find the surface area of the figure.

LA = # of lateral faces(area formula of lateral face)

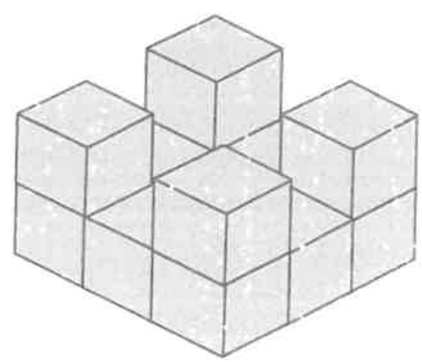
LA = ______________

B = area of base; B = __________ = __________

SA = LA + B = ______________

Example 2: Using Cubes

There are 13 cubes glued together forming the solid in the diagram. The edges of each cube are $\frac{1}{4}$ inch in length. Find the surface area of the solid.



Total number of faces: ____________________

Bottom: ______ Top: ______ Bottom sides: _______

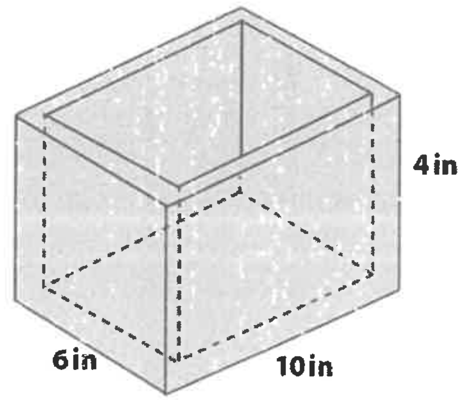
4 cubes minus the tops (since those have been counted already): ________

Area of 1 square: ____________________________________

Surface Area: ____________________________________

Example 3

Find the total surface area of the wooden jewelry box. The sides and bottom of the box are all $\frac{1}{4}$ inch thick.



What are the faces that make up this box?

How does this box compare to other objects that you have found the surface area of?

Large Prism

Small Prism

Surface Area of the Box

Lesson 23: The Volume of a Right Prism

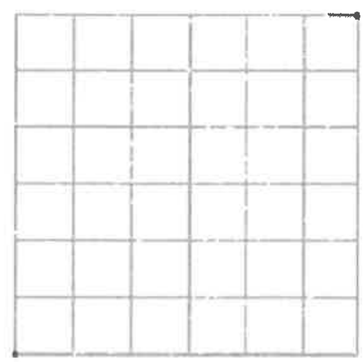
Classwork

Opening Exercise

Volume: A ____________________ given by the number of _________ ____________ needed to __________ the solid.

Most solids—rocks, baseballs, people—cannot be filled with unit cubes or assembled from cubes. Yet such solids still have volume. Fortunately, we do not need to assemble solids from unit cubes in order to calculate their volume. One of the first interesting examples of a solid that cannot be assembled from cubes, but whose volume can still be calculated from a formula, is a right triangular prism.

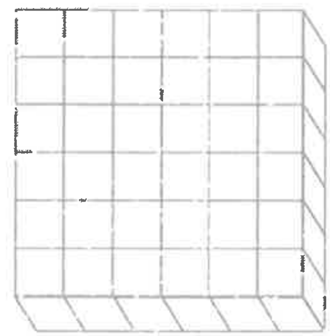
What is the area of the square pictured on the right? Explain.



Draw the diagonal joining the two given points; then, darken the grid lines within the lower triangular region. What is the area of that triangular region? Explain.

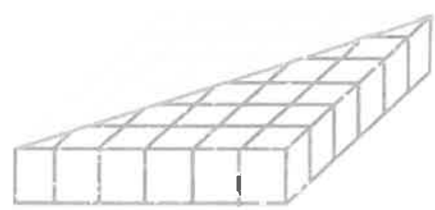
Exploratory Challenge: The Volume of a Right Prism

What is the volume of the right prism pictured on the right? Explain.

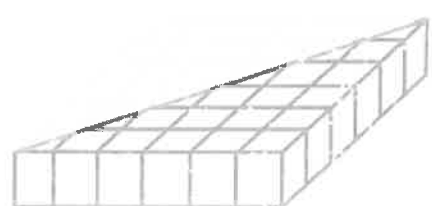


Draw the same diagonal on the square base as done before; then, darken the grid lines on the lower right triangular prism. What is the volume of that right triangular prism? Explain.

How could we create a right triangular prism with five times the volume of the right triangular prism pictured without changing the base? Draw your solution on the diagram, give the volume of the solid, and explain why your solution has five times the volume of the triangular prism.



What could we do to cut the volume of the right triangular prism pictured in half without changing the base? Draw your solution on the diagram, give the volume of the solid, and explain why your solution has half the volume of the given triangular prism.



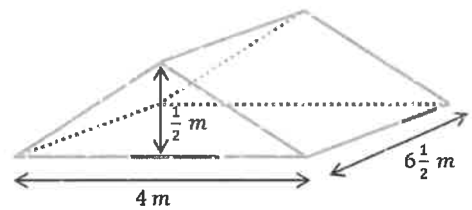
- The ____________________ of any right ________________ can be found by ________________________________ the

__________ of its ___________ times the ________________ of the prism.

To find the volume (V) of any right prism ... ______________________

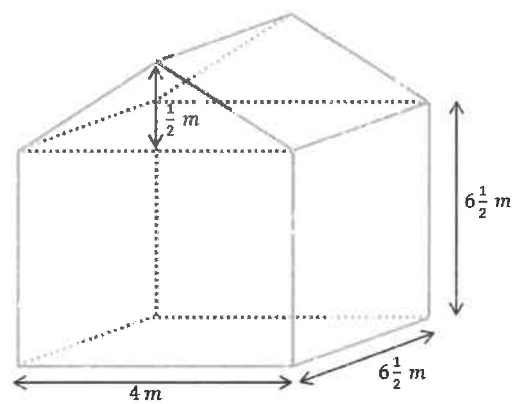
Example: The Volume of a Right Triangular Prism

Find the volume of the right triangular prism shown in the diagram using $V = Bh$.



Exercise: Multiple Volume Representations

1. The right pentagonal prism is composed of a right rectangular prism joined with a right triangular prism. Find the volume of the right pentagonal prism shown in the diagram.



2. Given a right rectangular prism with a volume of $10\frac{1}{2}$ in.^{3}, a length of 4 in., and a width of $2\frac{1}{2}$ in., find the height of the prism.

Lesson 24: The Volume of a Right Prism

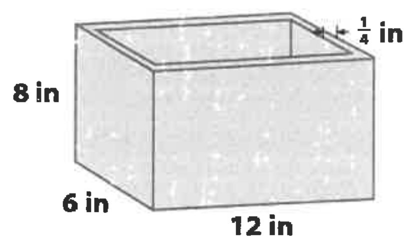
Classwork

Exploratory Challenge: Measuring a Container's Capacity

A box in the shape of a right rectangular prism has a length of 12 in, a width of 6 in, and a height of 8 in The base and the walls of the container are $\frac{1}{4}$ in. thick, and its top is open. What is the capacity of the right rectangular prism?

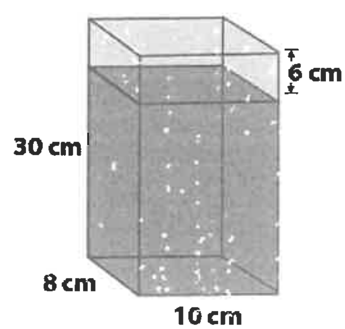
Capacity: Equal to the ____________________ of water needed to ___________ the prism to the ___________.

***When subtracting the thickness to get the new dimensions, you subtract ____________ the thickness of the ____________ and ___________, and _______ times the thickness of the ______________



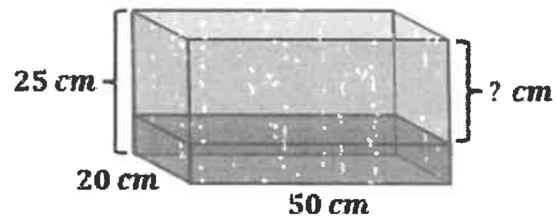
Example 1: Measuring Liquid in a Container in Three Dimensions

A glass container is in the form of a right rectangular prism. The container is 10 cm long, 8 cm wide, and 30 cm high. The top of the container is open, and the base and walls of the container are 3 mm (or 0.3 cm) thick. The water in the container is 6 cm from the top of the container. What is the volume of the water in the container?



Example 2

7.2 L of water are poured into a container in the shape of a right rectangular prism. The inside of the container is 50 cm long, 20 cm wide, and 25 cm tall. How far from the top of the container is the surface of the water? ($1\text{ L} = 1000\text{ cm}^3$)



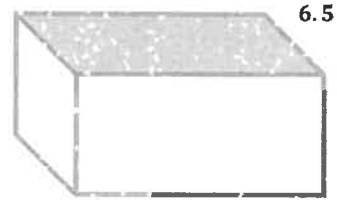
Example 3

A fuel tank is the shape of a right rectangular prism and has 27 L of fuel in it. It is determined that the tank is $\frac{3}{4}$ full. The inside dimensions of the base of the tank are 90 cm by 50 cm. What is the height of the fuel in the tank? How deep is the tank? ($1\text{ L} = 1{,}000\text{ cm}^3$)

Lesson 25: Volume and Surface Area

Warm-Up

What is the surface area and volume of the right rectangular prism?



Example 1: Volume of a Fish Tank

Jay has a small fish tank. It is the same shape and size as the right rectangular prism shown in the Opening Exercise.

a. The box it came in says that it is a 3-gallon tank. Is this claim true? Explain your reasoning. Recall that $1 \text{ gal} = 231 \text{ in}^3$.

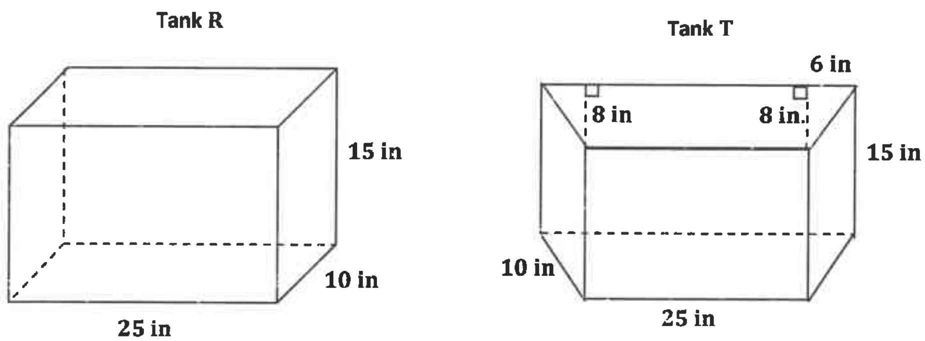
b. The pet store recommends filling the tank to within 1.5 in. of the top. How many gallons of water will the tank hold if it is filled to the recommended level?

c. Jay wants to cover the back, left, and right sides of the tank with a background picture. How many square inches will be covered by the picture?

d. Water in the tank evaporates each day, causing the water level to drop. How many gallons of water have evaporated by the time the water in the tank is four inches deep? Assume the tank was filled to within 1.5 in. of the top to start.

Exercise 1: Fish Tank Designs

Two fish tanks are shown below, one in the shape of a right rectangular prism (R) and one in the shape of a right trapezoidal prism (T).



a. Which tank holds the most water? Let $Vol(R)$ represent the volume of the right rectangular prism and $Vol(T)$ represent the volume of the right trapezoidal prism. Use your answer to fill in the blanks with $Vol(R)$ and $Vol(T)$.

______________ < ______________

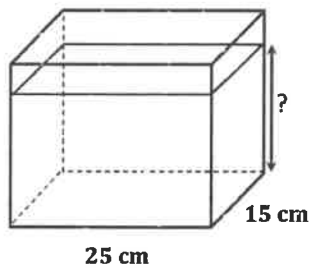
b. Which tank has the most surface area? Let $SA(R)$ represent the surface area of the right rectangular prism and $SA(T)$ represent the surface area of the right trapezoidal prism. Use your answer to fill in the blanks with $SA(R)$ and $SA(T)$.

______________ < ______________

c. Water evaporates from each aquarium. After the water level has dropped $\frac{1}{2}$ inch in each aquarium, how many cubic inches of water are required to fill up each aquarium? Show work to support your answers.

Practice:

1. A rectangular container 15 cm long by 25 cm wide contains 2.5 L of water.

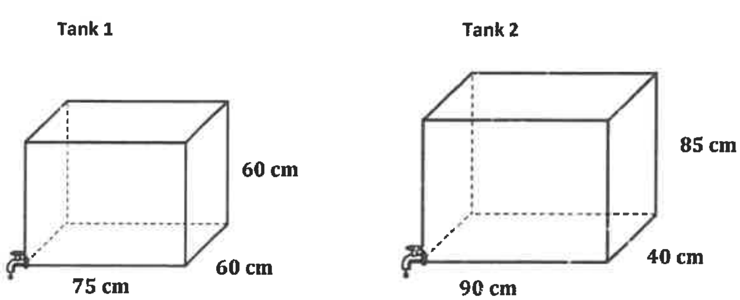


a. Find the height of the water level in the container. $(1\text{ L} = 1{,}000\text{ cm}^3)$

b. If the height of the container is 18 cm, how many more liters of water would it take to completely fill the container?

c. What percentage of the tank is filled when it contains 2.5 L of water?

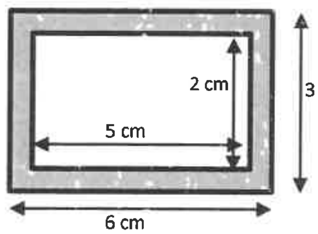
2. Two tanks are shown below. Both are filled to capacity, but the owner decides to drain them. Tank 1 is draining at a rate of **8** liters per minute. Tank 2 is draining at a rate of **10** liters per minute. Which tank empties first?



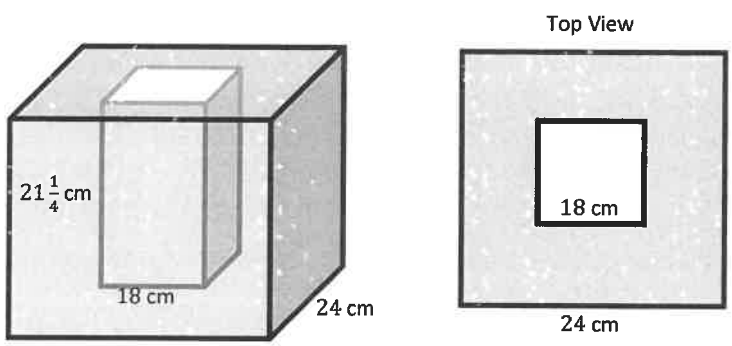
Lesson 26: Volume and Surface Area

Warm-Up

Calculate the area of the shaded region.



Example 1: Volume of a Shell



The insulated box shown is made from a large cube with a hollow inside that is a right rectangular prism with a square base. The figure on the right is what the box looks like from above.

a. Calculate the volume of the outer box.

b. Calculate the volume of the inner prism.

c. Describe in words how you would find the volume of the insulation.

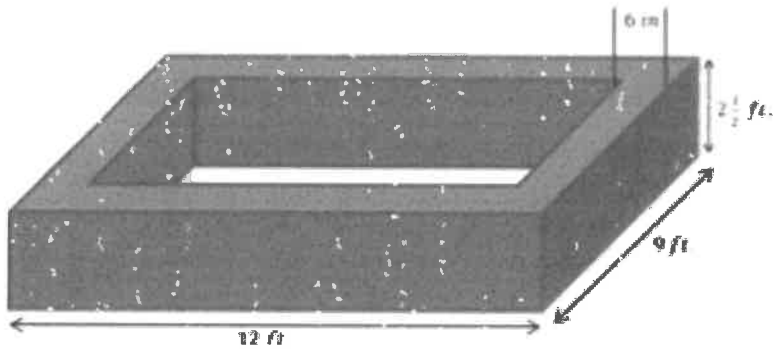
d. Calculate the volume of the insulation in cubic centimeters.

e. Calculate the amount of water the box can hold in liters.

($1\text{ cm}^3 = 1\text{ mL}$ *and* $1{,}000\text{ mL} = 1\text{ L}$)

Exercise 1: Brick Planter Design

You have been asked by your school to design a brick planter that will be used by classes to plant flowers. The planter will be built in the shape of a right rectangular prism with no bottom so water and roots can access the ground beneath. The exterior dimensions are to be $12\text{ ft.} \times 9\text{ ft.} \times 2\frac{1}{2}\text{ ft.}$ The bricks used to construct the planter are 6 in. long, $3\frac{1}{2}$ in. wide, and 2 in. high.

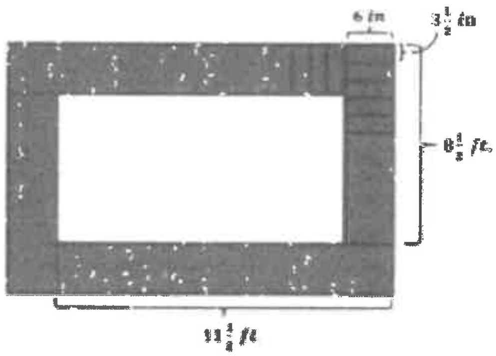


a. What are the interior dimensions of the planter if the thickness of the planter's walls is equal to the length of the bricks?

b. What is the volume of the bricks that form the planter?

c. If you are going to fill the planter $\frac{3}{4}$ full of soil, how much soil will you need to purchase, and what will be the height of the soil? **(The height of the soil will be $\frac{3}{4}$ of the $2\frac{1}{2}$ ft.)**

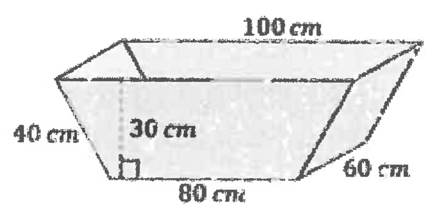
d. How many bricks are needed to construct the planter?



Exercise 2: Design a Feeder

You did such a good job designing the planter that a local farmer has asked you to design a feeder for the animals on his farm. Your feeder must be able to contain at least 100,000 cubic centimeters, but not more than 200,000 cubic centimeters of grain when it is full. The feeder is to be built of stainless steel and must be in the shape of a right prism but not a right rectangular prism. Sketch your design below including dimensions. Calculate the volume of grain that it can hold and the amount of metal needed to construct the feeder.

<u>$V = Bh$</u> <u>$SA = (LA - A_{top}) + 2B$; $LA = Ph$</u>



The feeder will require ____________ cm^2 of metal.

$1\ m^2 = 10{,}000\ cm^2$, so ____________ cm^2 = ________ m^2