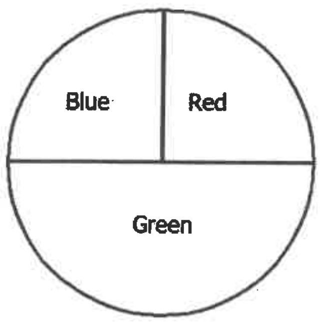
Name ______________________________ Date ____________ Period ______

Lesson 1: Chance Experiments

Example 1: Spinner Game

Suppose you and your friend are about to play a game using this spinner



Rules of the game:

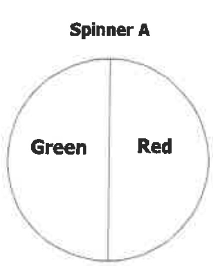
1. Decide who will go first.
2. Each person picks a color. Both players cannot pick the same color.
3. Each person takes a turn spinning the spinner and recording what color the spinner stops on. The winner is the person whose color is the first to happen 10 times.

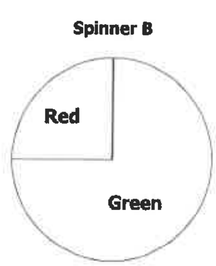
Play the game, and remember to record the color the spinner stops on for each spin.

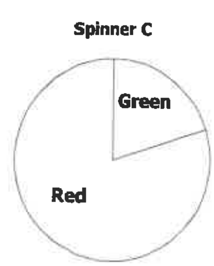
Exercises 1–4

1. Which color was the first to occur 10 times?
2. Do you think it makes a difference who goes first to pick a color?
3. Which color would you pick to give you the best chance of winning the game? Why would you pick that color?

4. Below are three different spinners. On which spinner is the green likely to win, unlikely to win, and equally likely to win?







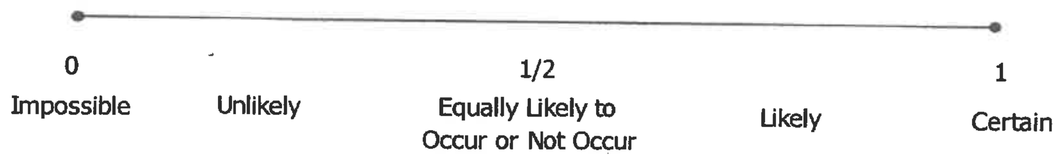
Example 2: What Is Probability?

Probability is a measure of how likely it is that an event will happen. A probability is indicated by a number between 0 and 1. Some events are certain to happen, while others are impossible. In most cases, the probability of an event happening is somewhere between certain and impossible.

For example, consider a bag that contains only red cubes. If you were to select one cube from the bag, you are certain to pick a red one. We say that an event that is certain to happen has a probability of 1. If we were to reach into the same bag of cubes, it is impossible to select a yellow cube. An impossible event has a probability of 0.

Description	Example	Explanation
Some events are *impossible*. These events have a probability of 0.	You have a bag with two green cubes, and you select one at random. Selecting a blue cube is an impossible event.	There is no way to select a blue cube if there are no blue cubes in the bag.
Some events are *certain*. These events have a probability of 1.	You have a bag with two green cubes, and you select one at random. Selecting a green cube is a certain event.	You will always get a green cube if there are only green cubes in the bag.
Some events are classified as *equally likely to occur or to not occur*. These events have a probability of $\frac{1}{2}$.	You have a bag with one blue cube and one red cube, and you randomly pick one. Selecting a blue cube is equally likely to occur or not to occur.	Since exactly half of the bag is made up of blue cubes and exactly half of the bag comprises red cubes, there is a 50/50 chance (equally likely) of selecting a blue cube and a 50/50 chance (equally likely) of NOT selecting a blue cube.
Some events are more likely to occur than not to occur. These events have a probability that is greater than 0.5. These events could be described as *likely* to occur.	If you have a bag that contains eight blue cubes and two red cubes and you select one at random, it is likely that you will get a blue cube.	Even though it is not certain that you will get a blue cube, a blue cube would be selected most of the time because there are many more blue cubes than red cubes.
Some events are less likely to occur than not to occur. These events have a probability that is less than 0.5. These events could be described as *unlikely* to occur.	If you have a bag that contains eight blue cubes and two red cubes and you select one at random, it is unlikely that you will get a red cube.	Even though it is not impossible to get a red cube, a red cube would not be selected very often because there are many more blue cubes than red cubes.

Probability Scale



Exercises 5–10

5. Decide where each event would be located on the scale above. Place the letter for each event in the appropriate place on the probability scale.

   Event:

   A. You will see a live dinosaur on the way home from school today.

   B. A solid rock dropped in the water will sink.

   C. A round disk with one side red and the other side yellow will land yellow side up when flipped.

   D. A spinner with four equal parts numbered 1–4 will land on the 4 on the next spin.

   E. Your full name will be drawn when a full name is selected randomly from a bag containing the full names of all of the students in your class.

   F. A red cube will be drawn when a cube is selected from a bag that has five blue cubes and five red cubes.

   G. Tomorrow the temperature outside will be −250 degrees.

6. Design a spinner so that the probability of spinning a green is 1.

7. Design a spinner so that the probability of spinning a green is 0.

8. Design a spinner with two outcomes in which it is equally likely to land on the red and green parts.

An event that is impossible has a probability of 0 and will never occur, no matter how many observations you make. This means that in a long sequence of observations, it will occur 0% of the time. An event that is certain has a probability of 1 and will always occur. This means that in a long sequence of observations, it will occur 100% of the time.

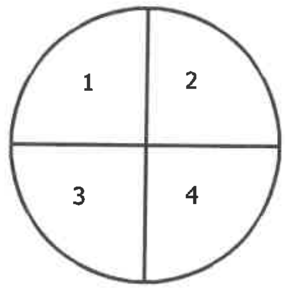
9. What do you think it means for an event to have a probability of $\frac{1}{2}$?

10. What do you think it means for an event to have a probability of $\frac{1}{4}$?

Lesson 2: Estimating Probabilities by Collecting Data

Exercises 1–8: Carnival Game

At the school carnival, there is a game in which students spin a large spinner. The spinner has four equal sections numbered 1–4 as shown below. To play the game, a student spins the spinner twice and adds the two numbers that the spinner lands on. If the sum is greater than or equal to 5, the student wins a prize.



Play this game with your partner 15 times. Record the outcome of each spin in the table below.

Turn	First Spin Results	Second Spin Results	Sum
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

1. Out of the 15 turns, how many times was the sum greater than or equal to 5?

2. What sum occurred most often?

3. What sum occurred least often?

4. If students were to play a lot of games, what fraction of the games would they win? Explain your answer.

5. Name a sum that would be impossible to get while playing the game.

6. What event is certain to occur while playing the game?

When you were spinning the spinner and recording the outcomes, you were performing a *chance experiment*. You can use the results from a chance experiment to estimate the probability of an event. In Exercise 1, you spun the spinner 15 times and counted how many times the sum was greater than or equal to 5. An estimate for the probability of a sum greater than or equal to 5 is

$$P(\text{sum } \geq 5) = \frac{\text{Number of observed occurrences of the event}}{\text{Total number of observations}}.$$

7. Based on your experiment of playing the game, what is your estimate for the probability of getting a sum of 5 or more?

8. Based on your experiment of playing the game, what is your estimate for the probability of getting a sum of exactly 5?

Example 2: Animal Crackers

A student brought a very large jar of animal crackers to share with students in class. Rather than count and sort all the different types of crackers, the student randomly chose 20 crackers and found the following counts for the different types of animal crackers. Estimate the probability of selecting a zebra.

Animal	Number Selected
Lion	2
Camel	1
Monkey	4
Elephant	5
Zebra	3
Penguin	3
Tortoise	2
	Total 20

Exercises 9–15

If a student randomly selected a cracker from a large jar:

9. What is your estimate for the probability of selecting a lion?

10. What is your estimate for the probability of selecting a monkey?

11. What is your estimate for the probability of selecting a penguin or a camel?

12. What is your estimate for the probability of selecting a rabbit?

13. Is there the same number of each kind of animal cracker in the jar? Explain your answer.

14. If the student randomly selected another 20 animal crackers, would the same results occur? Why or why not?

15. If there are 500 animal crackers in the jar, how many elephants are in the jar? Explain your answer.

Lesson 3: Chance Experiments with Equally Likely Outcomes

Exercises 1–6

Jamal, a seventh grader, wants to design a game that involves tossing paper cups. Jamal tosses a paper cup five times and records the outcome of each toss. An *outcome* is the result of a single trial of an experiment.

Here are the results of each toss:

Jamal noted that the paper cup could land in one of three ways: on its side, right side up, or upside down. The collection of these three outcomes is called the *sample space* of the experiment. The *sample space* of an experiment is the set of all possible outcomes of that experiment.

For example, the sample space when flipping a coin is heads, tails.

The sample space when drawing a colored cube from a bag that has 3 red, 2 blue, 1 yellow, and 4 green cubes is red, blue, yellow, green.

For each of the following chance experiments, list the sample space (i.e., all the possible outcomes).

1. Drawing a colored cube from a bag with 2 green, 1 red, 10 blue, and 3 black

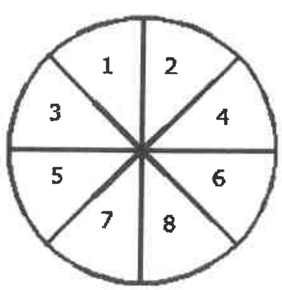
2. Tossing an empty soup can to see how it lands

3. Shooting a free throw in a basketball game

4. Rolling a number cube with the numbers 1–6 on its faces

5. Selecting a letter from the word *probability*

6. Spinning the spinner:



Example 2: Equally Likely Outcomes

The sample space for the paper cup toss was on its side, right side up, and upside down.

The outcomes of an experiment are equally likely to occur when the probability of each outcome is equal.

Toss the paper cup 30 times, and record in a table the results of each toss.

Toss	Outcome
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	
29	
30	

Exercises 7–12

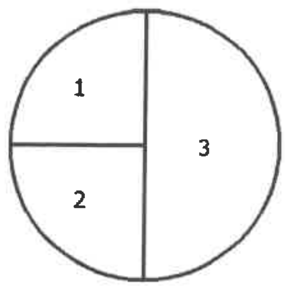
7. Using the results of your experiment, what is your estimate for the probability of a paper cup landing on its side?

8. Using the results of your experiment, what is your estimate for the probability of a paper cup landing upside down?

9. Using the results of your experiment, what is your estimate for the probability of a paper cup landing right side up?

10. Based on your results, do you think the three outcomes are equally likely to occur?

11. Using the spinner below, answer the following questions.



a. Are the events spinning and landing on 1 or 2 equally likely?

b. Are the events spinning and landing on 2 or 3 equally likely?

c. How many times do you predict the spinner will land on each section after 100 spins?

12. Draw a spinner that has 3 sections that are equally likely to occur when the spinner is spun. How many times do you think the spinner will land on each section after 100 spins?

Lesson 4: Calculating Probabilities for Chance Experiments with Equally Likely Outcomes

Examples: Theoretical Probability

In a previous lesson, you saw that to find an estimate of the probability of an event for a chance experiment you divide

$$P(\text{event}) = \frac{\text{Number of observed occurrences of the event}}{\text{Total number of observations}}.$$

Your teacher has a bag with some cubes colored yellow, green, blue, and red. The cubes are identical except for their color. Your teacher will conduct a chance experiment by randomly drawing a cube with replacement from the bag. Record the outcome of each draw in the table below.

Trial	Outcome
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

1. Based on the 20 trials, estimate for the probability of
   a. Choosing a yellow cube
   b. Choosing a green cube
   c. Choosing a red cube
   d. Choosing a blue cube

2. If there are 40 cubes in the bag, how many cubes of each color are in the bag? Explain.

3. If your teacher were to randomly draw another 20 cubes one at a time and with replacement from the bag, would you see exactly the same results? Explain

4. Find the fraction of each color of cubes in the bag.

Yellow

Green

Red

Blue

Each fraction is the *theoretical probability* of choosing a particular color of cube when a cube is randomly drawn from the bag.

When all the possible outcomes of an experiment are equally likely, the probability of each outcome is

$$P(\text{outcome}) = \frac{1}{\text{Number of possible outcomes}}.$$

An event is a collection of outcomes, and when the outcomes are equally likely, the theoretical probability of an event can be expressed as

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}.$$

The theoretical probability of drawing a blue cube is

$$P(\text{blue}) = \frac{\text{Number of blue cubes}}{\text{Total number of cubes}} = \frac{10}{40}.$$

5. Is each color equally likely to be chosen? Explain your answer.

6. How do the theoretical probabilities of choosing each color from Exercise 4 compare to the experimental probabilities you found in Exercise 1?

7. An experiment consisted of flipping a nickel and a dime. The first step in finding the theoretical probability of obtaining a heads on the nickel and a heads on the dime is to list the sample space. For this experiment, complete the sample space below.

Nickel Dime

What is the probability of flipping two heads?

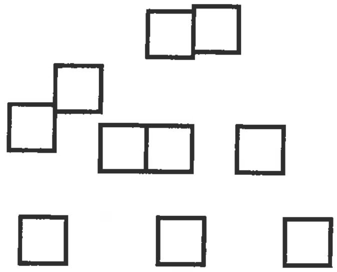
Exercises 1–4

1. Consider a chance experiment of rolling a six-sided number cube with the numbers 1–6 on the faces.
   a. What is the sample space? List the probability of each outcome in the sample space.
   b. What is the probability of rolling an odd number?
   c. What is the probability of rolling a number less than 5?

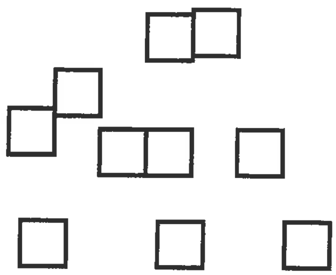
2. Consider an experiment of randomly selecting a letter from the word *number*.
   a. What is the sample space? List the probability of each outcome in the sample space.
   b. What is the probability of selecting a vowel?
   c. What is the probability of selecting the letter *z*?

3. Consider an experiment of randomly selecting a square from a bag of 10 squares.

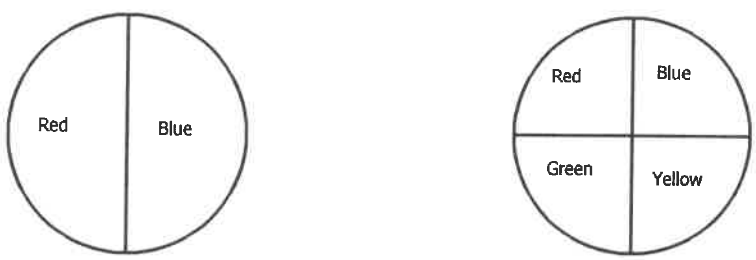
   a. Color the squares below so that the probability of selecting a blue square is $\frac{1}{2}$.



   b. Color the squares below so that the probability of selecting a blue square is $\frac{4}{5}$.



4. Students are playing a game that requires spinning the two spinners shown below. A student wins the game if both spins land on red. What is the probability of winning the game? Remember to first list the sample space and the probability of each outcome in the sample space. There are eight possible outcomes to this chance experiment.



Lesson Summary

When all the possible outcomes of an experiment are equally likely, the probability of each outcome is

$$P(\text{outcome}) = \frac{1}{\text{Number of possible outcomes}}.$$

An event is a collection of outcomes, and when all outcomes are equally likely, the theoretical probability of an event can be expressed as

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}.$$

Lesson 5: Chance Experiments with Outcomes That Are Not Equally Likely

In previous lessons, you learned that when the outcomes in a sample space are equally likely, the probability of an event is the number of outcomes in the event divided by the number of outcomes in the sample space. However, when the outcomes in the sample space are *not* equally likely, we need to take a different approach.

Example 1

When Jenna goes to the farmers' market, she usually buys bananas. The number of bananas she might buy and their probabilities are shown in the table below.

Number of Bananas	0	1	2	3	4	5
Probability	0.1	0.1	0.1	0.2	0.2	0.3

a. What is the probability that Jenna buys exactly 3 bananas?

b. What is the probability that Jenna does not buy any bananas?

c. What is the probability that Jenna buys more than 3 bananas?

d. What is the probability that Jenna buys at least 3 bananas?

e. What is the probability that Jenna does not buy exactly 3 bananas?

Notice that the sum of the probabilities in the table is one whole ($0.1 + 0.1 + 0.1 + 0.2 + 0.2 + 0.3 = 1$). This is always true; when we add up the probabilities of all the possible outcomes, the result is always 1. So, taking 1 and subtracting the probability of the event gives us the probability of something *not* occurring.

Exercises 1–2

Jenna's husband, Rick, is concerned about his diet. On any given day, he eats 0, 1, 2, 3, or 4 servings of fruits and vegetables. The probabilities are given in the table below.

Number of Servings of Fruits and Vegetables	0	1	2	3	4
Probability	0.08	0.13	0.28	0.39	0.12

1. On a given day, find the probability that Rick eats
   a. Two servings of fruits and vegetables

   b. More than two servings of fruits and vegetables

   c. At least two servings of fruits and vegetables

2. Find the probability that Rick does not eat exactly two servings of fruits and vegetables.

Example 2

Luis works in an office, and the phone rings occasionally. The possible number of phone calls he receives in an afternoon and their probabilities are given in the table below.

Number of Phone Calls	0	1	2	3	4
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{1}{9}$

a. Find the probability that Luis receives 3 or 4 phone calls.

b. Find the probability that Luis receives fewer than 2 phone calls.

c. Find the probability that Luis receives 2 or fewer phone calls.

d. Find the probability that Luis does not receive 4 phone calls.

Exercises 3–7

When Jenna goes to the farmers' market, she also usually buys some broccoli. The possible number of heads of broccoli that she buys and the probabilities are given in the table below.

Number of Heads of Broccoli	0	1	2	3	4
Probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{4}$	$\frac{1}{12}$

3. Find the probability that Jenna:

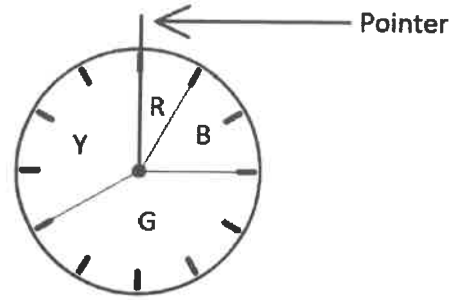
a. Buys exactly 3 heads of broccoli

b. Does not buy exactly 3 heads of broccoli

c. Buys more than 1 head of broccoli

d. Buys at least 3 heads of broccoli

The diagram below shows a spinner designed like the face of a clock. The sectors of the spinner are colored red (R), blue (B), green (G), and yellow (Y).



4. Writing your answers as fractions in lowest terms, find the probability that the pointer stops on the following colors.
   a. Red:

   b. Blue:

   c. Green:

   d. Yellow:

5. Complete the table of probabilities below.

Color	Red	Blue	Green	Yellow
Probability				

6. Find the probability that the pointer stops in either the blue region or the green region.

7. Find the probability that the pointer does not stop in the green region.

Lesson 6: Using Tree Diagrams to Represent a Sample Space and to Calculate Probabilities

Example 1: Two Nights of Games

Imagine that a family decides to play a game each night. They all agree to use a tetrahedral die (i.e., a four-sided pyramidal die where each of four possible outcomes is equally likely—see the image at the end of this lesson) each night to randomly determine if they will play a board game (B) or a card game (C). The tree diagram mapping the possible overall outcomes over two consecutive nights will be developed below.

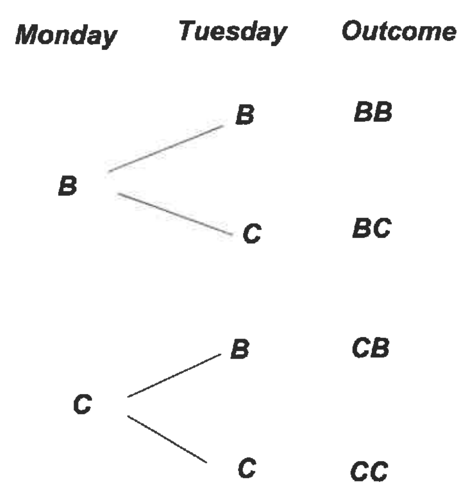
To make a tree diagram, first present all possibilities for the first stage (in this case, Monday).

Monday ***Tuesday*** ***Outcome***

B

C

Then, from *each* branch of the first stage, attach all possibilities for the second stage (Tuesday).



Note: If the situation has more than two stages, this process would be repeated until all stages have been presented.

a. If BB represents two straight nights of board games, what does CB represent?

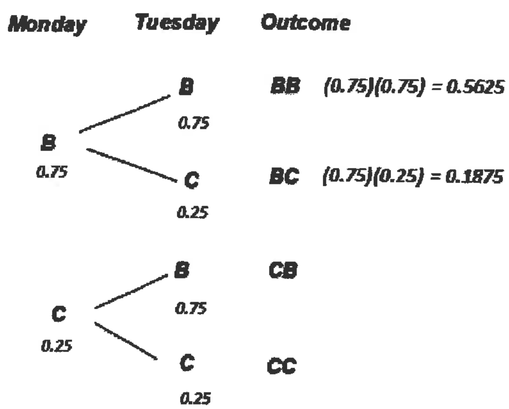
b. List the outcomes where exactly one board game is played over two days. How many outcomes were there?

Example 2: Two Nights of Games (with Probabilities)

In Example 1, each night's outcome is the result of a chance experiment (rolling the tetrahedral die). Thus, there is a probability associated with each night's outcome.

By multiplying the probabilities of the outcomes from each stage, we can obtain the probability for each "branch of the tree." In this case, we can figure out the probability of each of our four outcomes: BB, BC, CB, and CC.

For this family, a card game will be played if the die lands showing a value of 1, and a board game will be played if the die lands showing a value of 2, 3, or 4. This makes the probability of a board game (B) on a given night 0.75.



a. The probabilities for two of the four outcomes are shown. Now, compute the probabilities for the two remaining outcomes.

b. What is the probability that there will be exactly one night of board games over the two nights?

Exercises 1–3: Two Children

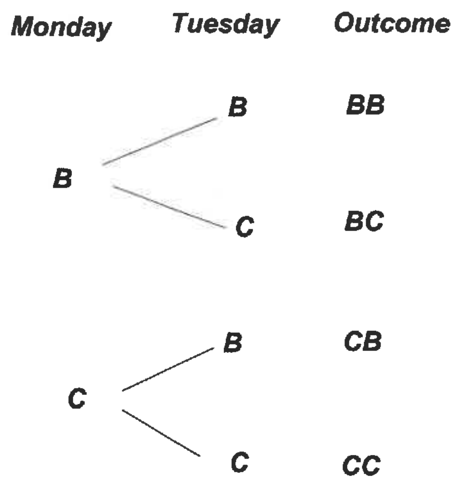
Two friends meet at a grocery store and remark that a neighboring family just welcomed their second child. It turns out that both children in this family are girls, and they are not twins. One of the friends is curious about what the chances are of having 2 girls in a family's first 2 births. Suppose that for each birth, the probability of a boy birth is 0.5 and the probability of a girl birth is also 0.5.

1. Draw a tree diagram demonstrating the four possible birth outcomes for a family with 2 children (no twins). Use the symbol B for the outcome of *boy* and G for the outcome of *girl*. Consider the first birth to be the first stage. (Refer to Example 1 if you need help getting started.)
2. Write in the probabilities of each stage's outcome to the tree diagram you developed above, and determine the probabilities for each of the 4 possible birth outcomes for a family with 2 children (no twins).
3. What is the probability of a family having 2 girls in this situation? Is that greater than or less than the probability of having exactly 1 girl in 2 births?

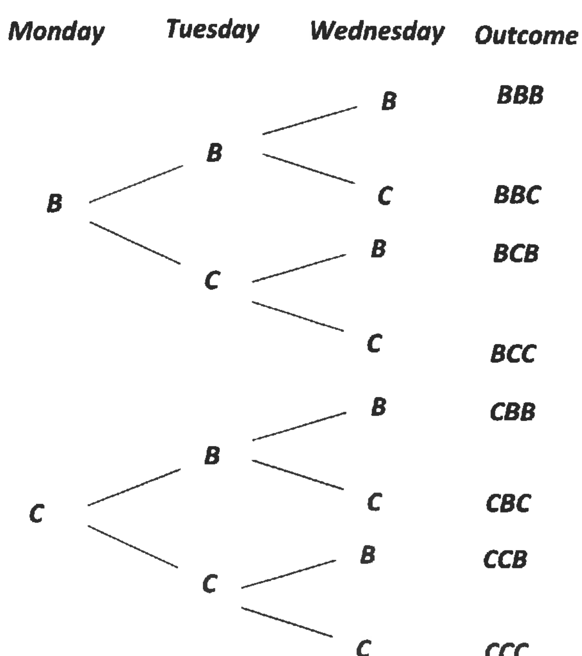
Lesson 7: Calculating Probabilities of Compound Events

Example 1: Three Nights of Games

Recall a previous example where a family decides to play a game each night, and they all agree to use a tetrahedral die (a four-sided die in the shape of a pyramid where each of four possible outcomes is equally likely) each night to randomly determine if the game will be a board (B) or a card (C) game. The tree diagram mapping the possible overall outcomes over two consecutive nights was as follows:



But how would the diagram change if you were interested in mapping the possible overall outcomes over three consecutive nights? To accommodate this additional third stage, you would take steps similar to what you did before. You would attach all possibilities for the third stage (Wednesday) to each branch of the previous stage (Tuesday).



Exercises 1–3

1. If BBB represents three straight nights of board games, what does CBB represent?

2. List all outcomes where exactly two board games were played over three days. How many outcomes were there?

3. There are eight possible outcomes representing the three nights. Are the eight outcomes representing the three nights equally likely? Why or why not?

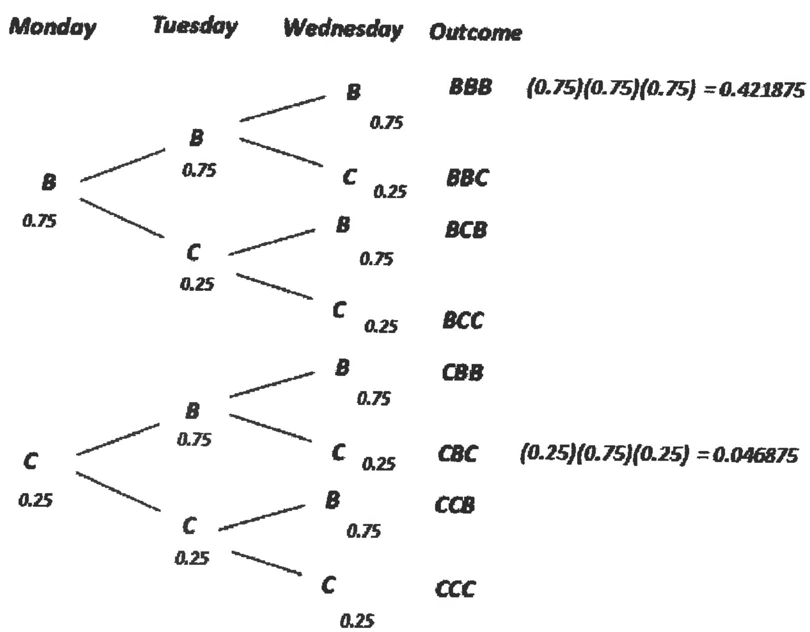
Example 2: Three Nights of Games (with Probabilities)

In Example 1, each night's outcome is the result of a chance experiment (rolling the four-sided die). Thus, there is a probability associated with each night's outcome.

By multiplying the probabilities of the outcomes from each stage, you can obtain the probability for each "branch of the tree." In this case, you can figure out the probability of each of our eight outcomes.

For this family, a card game will be played if the die lands showing a value of 1, and a board game will be played if the die lands showing a value of 2, 3, or 4. This makes the probability of a board game (B) on a given night 0.75.

Let's use a tree to examine the probabilities of the outcomes for the three days.



Exercises 4–6

4. Probabilities for two of the eight outcomes are shown. Calculate the approximate probabilities for the remaining six outcomes.

5. What is the probability that there will be exactly two nights of board games over the three nights?

6. What is the probability that the family will play at least one night of card games?

NAME:____________________________

Lesson 7 Skills & Problem-Solving Practice Notes

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

1. Rolling two number cubes and tossing one coin

2. Choosing rye or Bermuda grass and 3 different mixtures of fertilizer

3. Making a sandwich with ham, turkey, or roast beef; Swiss or provolone cheese; and mustard or mayonnaise

4. Tossing 4 coins

5. Choosing from 3 sizes of bottled water and from distilled, filtered, or spring water

6. Choosing from 3 flavors and 3 sizes of juice

7. Choosing from 35 flavors of ice cream; one, two, or three scoops; and sugar or waffle cone

8. Picking a day of the week and a date in the month of April

9. Rolling 3 number cubes and tossing 2 coins

10. Choosing a 4-letter password using only 5 letters that may each be used more than once

11. Choosing a bicycle with or without shock absorbers; with or without lights; and 5 color choices

12. A license plate that has 3 numbers from 0 to 9 and 2 letters where each number and a letter may be used more than once

13. Tradd owns 3 surfboards and 2 wet suits. If he takes one surfboard and one wet suit to the beach, how many different combinations can he choose?

NAME:________________________________

14. Trey is trying to decide which bag of dog food to buy. The brand he wants comes in 4 flavors and 3 sizes. How many choices are there?

15. To set a password, you must select 4 numbers from 0 to 9. How many possible passwords can be chosen if each number may be used more than once?

16. Margaret's favorite restaurant has 3 specials every day. There are 2 choices of vegetables and 3 choices of dessert. How many different meals could Margaret have if she chooses one special, one vegetable, and one dessert?

17. When Sunil goes to the building where he works, he can go through 4 different doors into the lobby. Then he can go to the seventh floor by taking 2 different elevators or 2 different stairways. How many different ways can Sunil get from outside the building to the seventh floor?

18. Jailin went to her local stereo store. Given her budget and the available selection, she can choose between 2 CD players, 5 amplifiers, and 3 pairs of speakers. How many different ways can Jailin choose one CD player, one amplifier, and one pair of speakers?

19. For dessert you can choose apple, cherry, blueberry, or peach pie to eat, and milk or juice to drink. How many different combinations of one pie and one beverage are possible?

20. Giorgio is taking a true or false quiz. There are six questions on the quiz. How many ways can the quiz be answered?

Exercises 7–10: Three Children

A neighboring family just welcomed their third child. It turns out that all 3 of the children in this family are girls, and they are not twins or triplets. Suppose that for each birth, the probability of a boy birth is 0.5, and the probability of a girl birth is also 0.5. What are the chances of having 3 girls in a family's first 3 births?

7. Draw a tree diagram showing the eight possible birth outcomes for a family with 3 children (no twins or triplets). Use the symbol B for the outcome of *boy* and G for the outcome of *girl*. Consider the first birth to be the first stage. (Refer to Example 1 if you need help getting started.)

8. Write in the probabilities of each stage's outcomes in the tree diagram you developed above, and determine the probabilities for each of the eight possible birth outcomes for a family with 3 children (no twins or triplets).

9. What is the probability of a family having 3 girls in this situation? Is that greater than or less than the probability of having exactly 2 girls in 3 births?

10. What is the probability of a family of 3 children having at least 1 girl?

Lesson Summary

The use of tree diagrams is not limited to cases of just two stages. For more complicated experiments, tree diagrams are used to organize outcomes and to assign probabilities. The tree diagram is a visual representation of outcomes that involve more than one event.