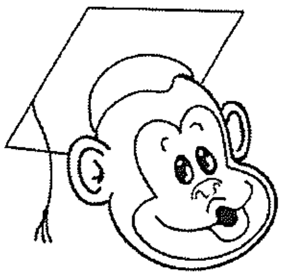
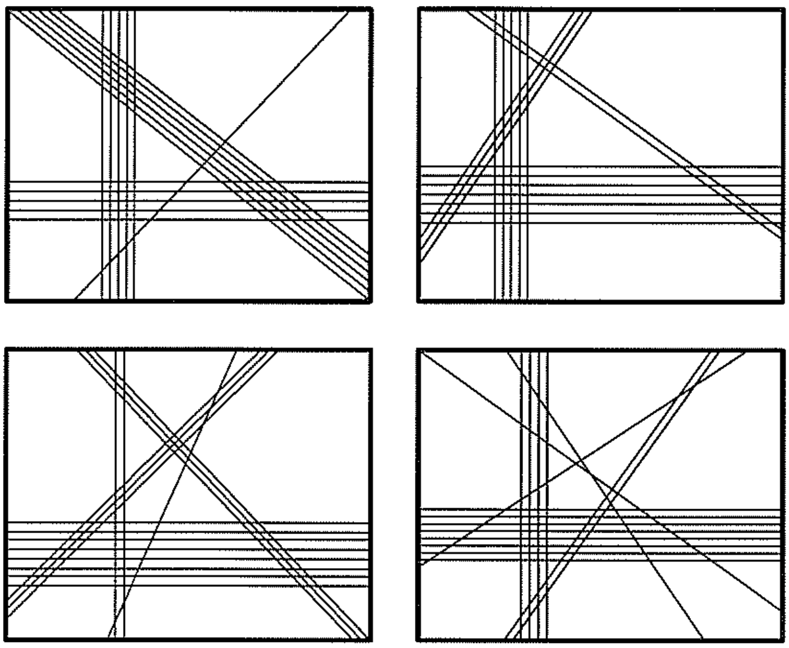
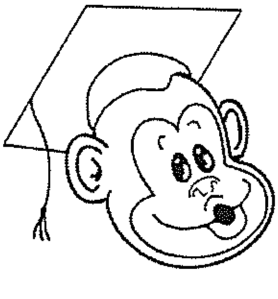


Solution: See images below.

Below are the four families of solutions that Fourier found. The upper left solution shows a set of six parallel lines, two sets of five parallel lines, and a single line. Taking all possible combinations of these sets of lines gives a total of (6 × 5) + (6 × 5) + (6 × 1) + (5 × 5) + (5 × 1) + (5 × 1) = 30 + 30 + 6 + 25 + 5 + 5 = 101 intersection points.

Similarly, the upper right solution demonstrates that 101 = 21 + 35 + 14 + 15 +10 + 6; the lower left solution demonstrates that 101 = 6 + 3 + 6 + 9 + 3 + 24 + 16 + 8 + 24 + 2; and the lower right solution demonstrates that 101 = 4 + 4 + 4 + 1 + 1 + 1 + 2 + 4 + 32 + 16 + 8 + 8 + 8 + 8.



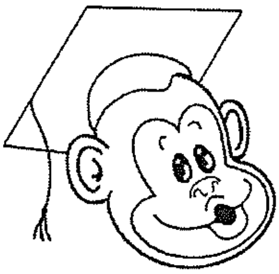


Solution: 79,365.

One way to tackle the problem is to find the first number consisting of all 1's (called a *repunit*) that is divisible by 7. To do that, just set up a standard long division, adding as many 1's as you need until you get a remainder of zero. You'll get the equation 111,111 = 7 × 15,873, and because this equation is the smallest of its kind, we can multiply both sides by 5 to yield 555,555 = 7 × 79,365.

You can also multiply backwards to find the answer. That is, what number times 7 gives a 5 as the units digit in the result? Obviously, that's 5. But then 5 × 7 = 35, so a 3 carries to the tens column. So then, what number times 7, and then adding the carried 3, gives 5? That's 6, since 6 × 7 + 3 = 45. That gives a 4 that carries… and so on. This process will eventually lead to the answer, too.

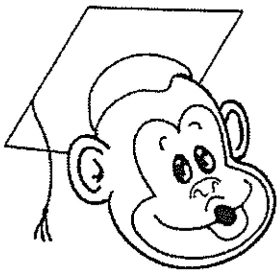
Of course, the method that requires the least amount of thinking is to just use a calculator. Enter a string of 5's, divide by 7, and keep increasing the number of 5's until the answer is an integer.



Solution: the 2080s.

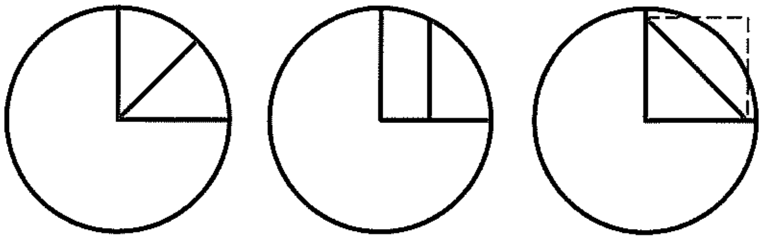
Both 2200 and 110 are divisible by 11. Consequently, 2200 – 110 = 2090 is also divisible by 11, so that 2090 is one of the years in the 21st century that is divisible by 11. The last year before 2090 divisible by 11 is 2090 – 11 = 2079. If 2079 and 2090 are both divisible by 11, then none of the years in between — 2080, 2081, 2082, ..., 2089 — are divisible by 11. Consequently, the 2080's are the decade you're looking for.

You could have also approached this one from the other direction. Since 2000 ÷ 11 = 181.81, then 11 × 182 = 2002 is the first year in the 21st century divisible by 11. Then just keep adding 11 to show that 2013, 2024, 2035, 2046, 2057, 2068, 2079, and 2090 are also multiples of 11. From this list, you can see that all years in the 2080's were skipped.



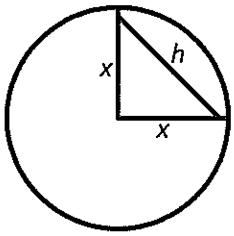
Solution: middle, shortest, longest.

The length of the segment on the left is simply the radius of the circle, and the segment in the second circle is clearly shorter than the radius.



So the question is, is the segment in the third circle longer or shorter than the other two? If you think of the segment in the third circle as the diagonal of a square, that square must extend beyond the circle for the segment to bisect the area. Consequently, the diagonal must be greater than the radius of the circle, so the third segment is the longest.

But perhaps that picture doesn't convince you. A more rigorous proof is to express the length of the segment in terms of the radius. Let x represent the lengths of the legs of the triangle, and h is the length of the segment in question.



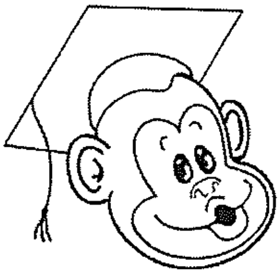
The area of the triangle is $A = \frac{1}{2}x^2$. But its area is also $A = \frac{1}{2}(\frac{1}{4}\pi r^2)$, because the triangle is one of two equal halves in the quarter-circle. This leads to an equation that can be solved for x in terms of r.

$$\tfrac{1}{2}x^2 = \tfrac{1}{2}\left(\tfrac{1}{4}\pi r^2\right)$$
$$x^2 = \tfrac{1}{4}\pi r^2$$
$$x = r\sqrt{\tfrac{\pi}{4}}$$

The hypotenuse of the triangle, h, is the segment about which we are concerned. Its length can now be found with the Pythagorean theorem.

$$h^2 = x^2 + x^2 = 2x^2$$
$$h^2 = 2\left(r\sqrt{\tfrac{\pi}{4}}\right)^2$$
$$h = \tfrac{\pi}{2}r \approx 1.57r$$

Because $h > r$, the third segment is greater than the radius, so it is the longest.

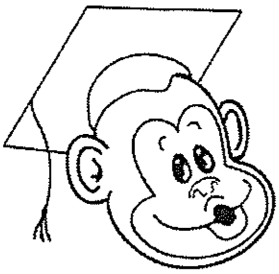


Solution: 45.

The example in the puzzle showed that 9 can be written as a sum of consecutive integers in three different ways. A little investigating reveals that 10, 11, 12, 13, and 14 can each be written as a sum of consecutive integers in two different ways, but 15 can be written as a sum of consecutive integers in four different ways. So this leads us to ask — how are 9 and 15 different from 10, 11, 12, 13, and 14, and how are they different from each other?

The difference is the number of odd divisors. The number 9 has three odd divisors (1, 3, and 9), the numbers 10, 11, 12, 13, and 14 each have two odd divisors, and 15 has four odd divisors (1, 3, 5, and 15). In general, the number of possible sums for a given positive integer n equals the number of odd divisors of n. The number 45 has six odd divisors, namely 1, 3, 5, 9, 15, and 45, so it can be represented as the sum of consecutive positive integers in six different ways. As it turns out, 45 is the smallest number with six odd divisors. (It is left to you to prove that there are no smaller numbers with exactly six odd divisors.)

The connection between odd divisors and sums may not be immediately obvious, but notice that 3 is an odd divisor of 45, and if you divide 45 by 3 to get 15, you're at the middle of a three-term sequence of consecutive numbers adding up to 45; that is, $45 \div 3 = 15$ leads to the sum $14 + 15 + 16$. Similarly, $45 \div 1 = 45$ leads to a one-term sum with 45 as the middle number; $45 \div 5 = 9$ leads to a five-term sum with 9 as the middle number, $7 + 8 + 9 + 10 + 11$; and, of course, $45 \div 9 = 5$ leads to a nine-term sum with 5 as the middle number, $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$. The other two sums result from two times an odd divisor, in this case $45 \div (1 \times 2) = 22.5$ leads to the two-term sum $22 + 23$, and $45 \div (3 \times 2) = 7.5$ leads to the six-term sum $5 + 6 + 7 + 8 + 9 + 10$.



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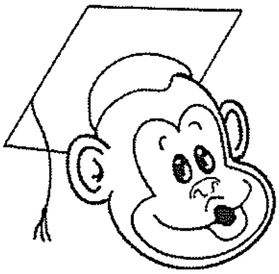


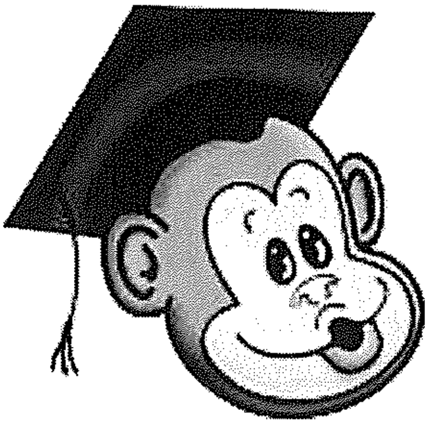
Solution: 48, 80.

The smallest number with 10 divisors is $48 = 2^4 \times 3$.

In general, the number $p^a \times q^b$ has $(a+1)(b+1)$ divisors if p and q are prime numbers. Because 2 and 3 are the smallest prime numbers, a number of the form $2^n \times 3$ is the smallest number with $2(n + 1)$ divisors, as follows:

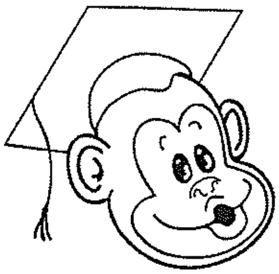
n	$2^n \times 3$	Number of divisors of $2^n \times 3$
0	3	2
1	6	4
2	12	6
3	24	8
4	48	10

From this pattern, it isn't hard to conclude that the next number with precisely 10 divisors can be obtained by replacing 3 with the next prime number, 5, to get $2^4 \times 5 = 80$.



Solution: x=3 and y=2.

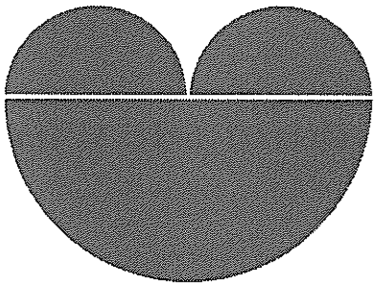
Adding and subtracting the equations we see that the numbers become 10,000m 10,000, and 50,000; and 3,502, -3.502, and 3,502. Dividing by 10,000, and by 3,502, we obtain:
x + y =5 and x – y = 1. Anyone can solve such simple equations in his head.

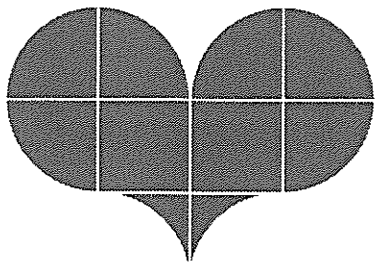


Solution: Answers will vary.

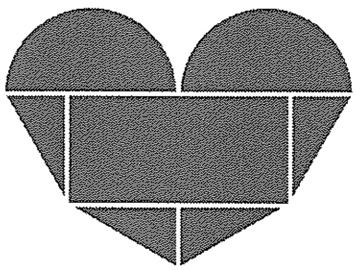
It's possible to construct a heart in many different ways using different numbers of shapes, though some of them look better than others.

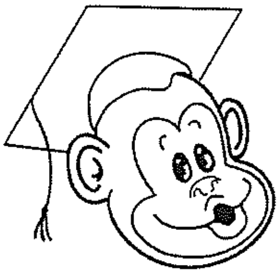
A heart using just three shapes can be made from two small semicircles and one large semicircle (below left), but this heart is very round and doesn't look as good as others. A more elaborate heart can be made with ten shapes (below right).





The heart below, which may look the most like a traditional heart shape, can be constructed with seven pieces.



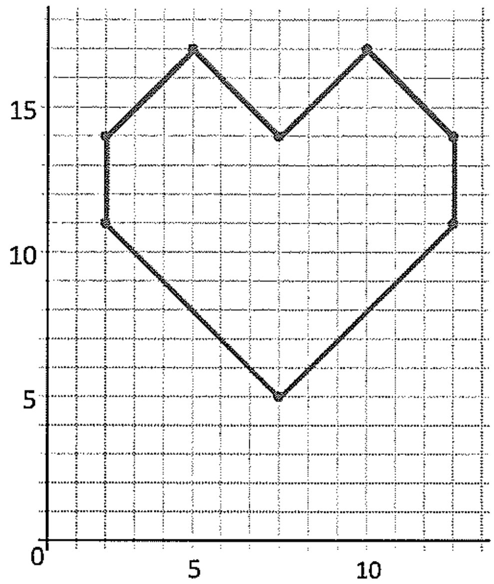


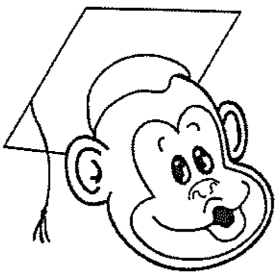
Solution: $n = 14$.

The coordinates include $n + 3$, n, $n - 3$, $n - 6$, $n - 9$, and $n - 12$. Because (2, 14) is one of the points, then 2 must be the smallest value from that list, which is $n - 12$. Since $n - 12 = 2$, then $n = 14$.

Note that all of the other coordinates have larger values. The next smallest value is $n - 9$, but if $n - 9 = 2$, then $n - 12 = 2 - 3 = -1$. This is impossible, because all coordinates have positive integer values. All of the other coordinates have values greater than $n - 9$, so none of them can be 2, so $n - 12 = 2$, as stated above.

When the points are plotted on a coordinate graph, the following heart-shaped figure results:





Solution: Answers will vary.

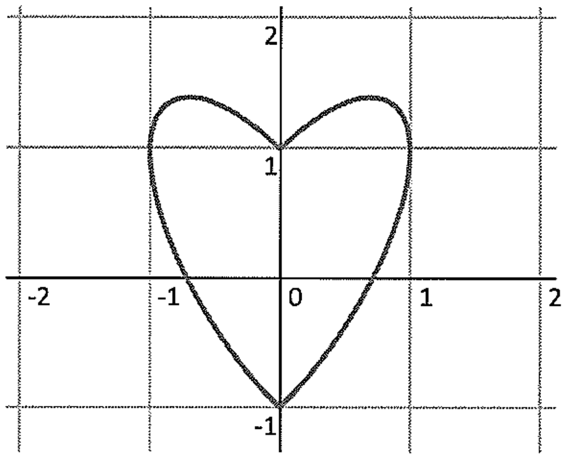
One way to create a heart is to use the graphs of two ellipses and restrict the domain.

$$2x^2-2xy+y^2-1=0 \quad \{x \mid x\geq 0\}$$
$$2x^2+2xy+y^2-1=0 \quad \{x \mid x\leq 0\}$$

Alternatively, you can solve for *y* and use the absolute value function, and then restricting the domain is unnecessary.

$$y=|x|+\sqrt{1-x^2}$$
$$y=|x|-\sqrt{1-x^2}$$

In either case, the result is a heart curve that looks like this:



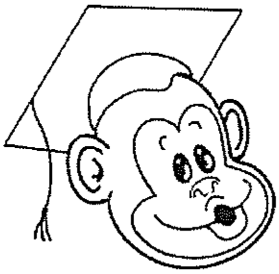
To move heart so that it covers (2,14), adjust the absolute value equations as follows:

$$y=|x-2|+\sqrt{1-(x-2)^2}+14$$
$$y=|x-2|-\sqrt{1-(x-2)^2}+14$$

The following polar equation will yield a cardioid:

$$r=1-\sin\theta$$

Other heart-shaped graphs can be created using polar, parametric or rectangular equations. A number of examples can be found at http://mathworld.wolfram.com/HeartCurve.html.



Solution: 25 units.

The following table shows some Pythagorean triples and several of their multiples:

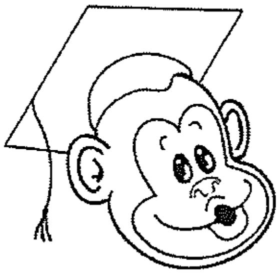
Pythagorean Triple	Multiples
(3, 4, 5)	(6, 8, 10)
	(9, 12, 15)
	(12, 16, 20)
	(15, 20, **25**)
(5, 12, 13)	(10, 24, 26)
	(15, 36, 39)
	(20, 48, 52)
(7, 24, **25**)	(14, 48, 50)
	(21, 72, 75)
(9, 40, 41)	(18, 80, 82)

Notice that the fourth multiple of (3, 4, 5) is (15, 20, 25), which has a hypotenuse of 25 units. Similarly, the primitive triple (7, 24, 25) also has a hypotenuse of 25 units.

There are several methods for generating Pythagorean triples. One is to take any odd number, square it, and represent it as the sum of consecutive integers. For instance, $7^2 = 49$, and $49 = 24 + 25$, so (7, 24, 25) is a Pythagorean triple.

Algebraically, if the hypotenuse is $k + 1$ units and the longer leg is k units, then the shorter leg will be $\sqrt{2k+1}$ units. The following shows that these values satisfy the Pythagorean theorem:

$$\left(\sqrt{2k+1}\right)^2 + k^2 = (k+1)^2$$
$$(2k+1) + k^2 = k^2 + 2k + 1$$
$$k^2 + 2k + 1 = k^2 + 2k + 1$$

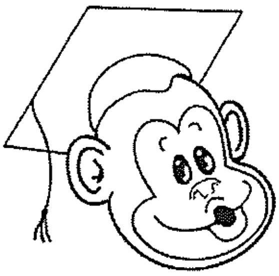


Solution: May and June.

To see how this works, all you need to know is that months with 30 days push the first of the month ahead two days of the week, whereas months with 31 days push ahead three days. For example, March has 31 days, so if March 1 is a Tuesday, then April 1 is a Friday (move ahead three days of the week). Similarly, April has 30 days, so May 1 will be a Sunday (move ahead just two days of the week).

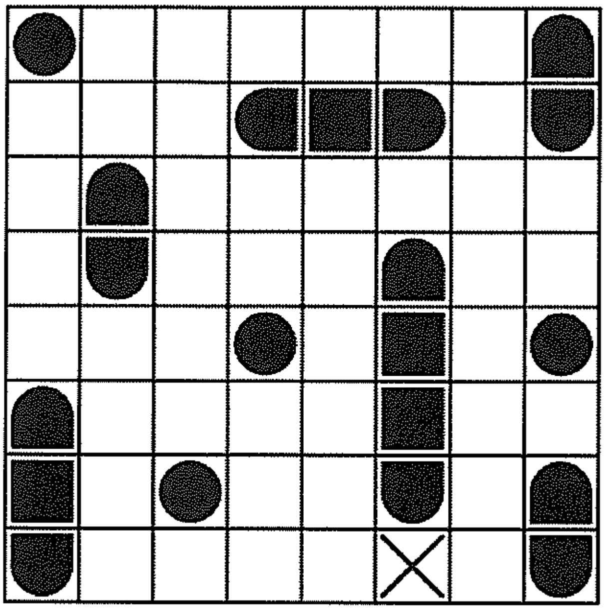
The table below summarizes what will happen in leap and non-leap years. August does not share the same day with any other month in a non-leap year, and October does not share the same day with any other month in a leap year — but only May and June never share the same day of the week with any other months, whether it's a leap year or not.

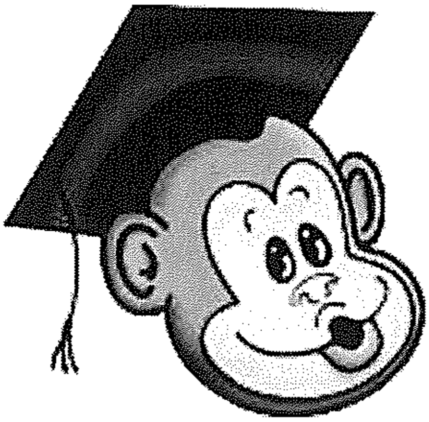
Month	**Months with Same First Day in a Non-Leap Year**	**Months with Same First Day in a Leap Year**
January	October	April, July
February	March, November	August
March	February, November	November
April	July	January, July
May	None	None
June	None	None
July	April	January, April
August	None	February
September	December	December
October	January	None
November	February, March	March
December	September	September



Solution:

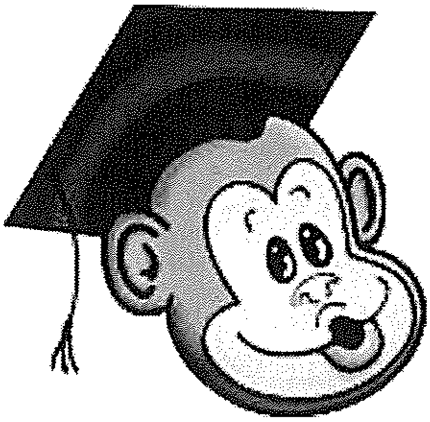
The 10 ships are arranged in the grid as shown below.





Solution: About 10 inches.

The distance from one knothole to another cut of the same knothole is about two-thirds the width of the whole plywood sheet or 30 inches. The diameter of the log is $30/\pi$ = about 10 inches.



Solution: Yes, *C* and *D* turn clockwise and *B* turns counterclockwise. The wheels can also turn if all 4 belts are crossed, but not if 1 or 3 belts are.